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Obtaining Geostationary Orbit from an Off-Nominal Low Transfer Orbit

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A theory for optimal apogee raising maneuvers for geostationary spacecraft injected in a low transfer orbit and equipped with a solid propellant apogee boost motor is developed. These maneuvers are made by hydrazine thrusters before the apogee boost motor firing. The decrease in propellant needed for station acquisition amounts to about 15-19 m/s for each 1000 km the apogee is below nominal compared with the solution of only using these thrusters after the firing.

Nomenclature

ABM = apogee boost motor
a = semimajor axis of TO
C = exhaust velocity of the ABM
c = exhaust velocity of the RCS
E = eccentric anomaly
e = eccentricity
f = acceleration caused by the thrust
i_0 = inclination change caused by pulse Δv_0
i_1 = inclination change caused by pulse Δv_1
i_2 = inclination change caused by pulse Δv_2
i = $i_0 + i_1 + i_2$
m_0 = initial spacecraft mass before maneuvers
m_1 = mass before ABM firing
m_2 = mass after ABM firing
m_3 = mass with no propellant left
NSO = near synchronous orbit, the orbit the spacecraft is injected into by the ABM firing
p = $a(1-e^2)$
RCS = reaction control system
SO = geosynchronous orbit
TO = transfer orbit, the orbit the spacecraft is injected into by the launcher
v_0 = perigee velocity in initial TO
v_1 = perigee velocity in TO after apogee raising
v_2 = apogee velocity in TO after apogee raising
v_3 = velocity in NSO after pulse Δv_1 (at an apse)
v_4 = velocity in NSO before pulse Δv_2 (at the other apse)
v_5 = geostationary velocity
w = argument of perigee
x = amount of apogee raising
α_0 = angle between velocity v_0 and pulse Δv_0
α_1 = angle between velocity v_1 and pulse Δv_0
α_2 = angle between velocity v_2 and pulse Δv_1
α_3 = angle between velocity v_3 and pulse Δv_1
α_4 = angle between velocity v_4 and pulse Δv_2
α_5 = angle between velocity v_5 and pulse Δv_2
ΔV = total propulsion capacity of ABM + RCS
Δv_0 = pulse used for apogee raising in TO
Δv_1 = pulse at apogee of the TO to raise perigee to geostationary altitude
Δv_{10} = Δv_1 for the nominal transfer with TO apogee at geostationary altitude
Δv_2 = pulse to circularize the orbit half a revolution after the pulse Δv_1
Δv_{ABM} = pulse caused by the ABM firing. If Δv_0 is zero this pulse is equal to Δv_{10}
Δv_E = increase of the total capacity of the propulsion system of the spacecraft (ABM + RCS) obtained by using some hydrazine before the ABM firing
θ = true anomaly
μ = gravitational constant

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Introduction

THE SMS-A spacecraft launched from the Eastern Test Range on May 17, 1974 did not reach its nominal transfer orbit with apogee near geostationary altitude; the obtained apogee altitude was about 3500 km below nominal.[1] The reason for this anomaly was a rupture in the flexible lines supplying the first stage main engine of its Delta 2914 launch vehicle with liquid oxygen. This caused the first stage to underperform and the velocity of the spacecraft at first stage cutoff was about 200 m/s lower than nominal. The onboard inertial guidance system preprogrammed for the nominal flight path compensated this deficiency by letting the second stage burn longer than nominal before cutoff (SECO 1) and the nominal coast orbit was obtained. But as a result of this, the remaining propulsion capacity (second and third stages) was insufficient for the final injection into transfer orbit; the third stage ran out of fuel before the preprogrammed velocity increment had been given.

In a similar way, most minor deficiencies of the propulsion system of the launcher should lead to a transfer orbit with a too low apogee but with essentially nominal perigee altitude and orientation (i.e., inclination, ascending node, and argument of perigee) provided that the inertial guidance system is working correctly.

There are, therefore, good reasons to consider a transfer orbit anomaly of this type as one of the most likely in the class of non-nominal transfer orbits from which it is at all possible to reach the geostationary orbit using the limited capacity of the onboard propulsion system (ABM + hydrazine thrusters).

Prior to launch of the first geostationary spacecraft of ESA (GEOS 1 on April 4, 1977), the transfer scheme described in this paper was discovered and software for the computations needed in such a scheme was included in the MSSS (multisatellite support system) of ESOC (European Space Operations Centre).

Effect for the Total Propulsion Capacity of Using the Hydrazine RCS Before the ABM Firing

From the well-known rocket equation follows that the total delta-v obtainable by using the ABM and the hydrazine RCS is

$$\Delta V = c \ln(m_0/m_1) + C \ln(m_1/m_2) + c \ln(m_2/m_3) \qquad (1)$$

It is clear that $m_0 - m_1$ is the amount of hydrazine used before the ABM firing and that $m_1 - m_2$ is the propellant mass of the ABM. Considering m_1 as a free parameter, the derivative of ΔV with respect to m_1 is

$$\frac{d\Delta V}{dm_1} = (C-c)\left(\frac{1}{m_1} - \frac{1}{m_2}\right) \tag{2}$$

as, of course, $dm_2/dm_1 = 1$, while m_0 and m_3 do not depend on m_1. As $C > c$ (about 2.8 km/s, respectively, 2.1 km/s) and $m_1 > m_2$, $d\Delta V/dm_1$, is the entity that is negative and ΔV is a decreasing function of m_1 and consequently an increasing function of the amount of hydrazine used before the ABM firing $(m_0 - m_1)$.

A sufficient condition for one injection scheme to require less propellant than another is consequently that both of the following two conditions are fulfilled: 1) the transfer requires less delta-v; and 2) a larger amount of hydrazine is used before the apogee boost motor firing.

Optimal Apogee Raising Maneuvers

For the type of non-nominal TO discussed in the introduction the apsidal line is in or at least close to the target plane of the SO, while the apogee radius is smaller than the radius of the geostationary orbit.

The optimal transfer (minimal delta-v) from such an orbit to the target SO in the absence of constraints from the propulsion system is well-known from the general theory of optimal transfers.[2]

It is a two impulse generalized Hohmann transfer; the first pulse being made at perigee to raise the apogee to the altitude of the circular orbit and to slightly decrease the inclination to the target plane, and the second pulse being given at apogee to circularize the orbit and to rotate the orbital plane around the apsidal line into the desired orientation. This can be thought of as a continuation of the originally foreseen transfer scheme, the lack of propulsion capacity of the launcher being compensated for by operating the RCS at a perigee passage an integer number of revolutions after the injection into TO.

Such a transfer scheme is not possible, however, with a noncutoff solid propellant apogee boost motor loaded for the apogee pulse of the nominal mission, as the decrease of mass will cause the ABM to give a velocity increment too large to fit into this transfer scheme.

To study the situation closer a one parameter family of transfers to the geostationary orbit will be analyzed (Fig. 1).

The transfer scheme is started with a perigee pulse Δv_0 raising the apogee with an amount x. Then follows a pulse Δv_1 at apogee raising the perigee to geostationary altitude and finally a pulse Δv_2 after half a revolution circularizing the orbit.

These three pulses will then be made on the apsidal line of the original TO assumed to coincide (being close to) the line of nodes between the planes of the TO and the SO. The necessary orbit plane change is assumed to be partitioned in an optimal way between the pulses that consequently are all inclined to the velocity vector, although orthogonal to the radius vector (this point will be analyzed more closely below).

The optimal transfer scheme discussed above corresponds to the case of choosing x as the total apogee default in which case the pulse Δv_2 vanishes. The transfer scheme that starts by firing the ABM without any previous corrections of the TO corresponds to the case of x being zero.

That the latter statement is correct can be seen in the following way: In case of a low transfer orbit with apsidal line in the plane of the target orbit, firing the optimal ABM is made at apogee in a direction orthogonal to the radius vector and inclined to the apogee velocity vector rotating the orbital plane around the nodal line towards the plane of the target SO. The resulting NSO will have a much smaller eccentricity and inclination than the TO but the direction of the apsidal line and the nodal line relative to the target SO plane will be unchanged and the perigee will be below geostationary altitude. We are therefore again in the situation discussed in the beginning of this paragraph. As the transfer no more involves a solid propellant ABM but only the hydrazine RCS allowing for any size of pulses, the optimal transfer quoted there can be used. But as the NSO is almost circular (otherwise the SO is completely out of reach using the hydrazine propulsion system), the penalty for reversing the order of the pulses is negligible, and to simplify the following discussion it will be assumed that the pulse at the apse of the ABM firing is made first. The compound effect on the orbit of the ABM pulse and this first pulse an integer number of NSO revolutions later is then equal to the effect of the vector sum of these two pulses. This vector sum is just the pulse Δv_1 quoted above, while the second pulse in the NSO is the pulse Δv_2.

The partition of the necessary orbit plane change between the two pulses Δv_1 and Δv_2 (Δv_0 being zero) is of course again to be optimal.

The optimal partition of the orbit plane change between the three pulses Δv_0, Δv_1, and Δv_2 is quite straightforward but will be touched upon shortly. Figure 2 illustrates the velocity parallelogram for any of the three impulsive velocity changes of the transfer scheme as seen along radius vector. While the magnitudes of the velocities before and after each of the pulses are determined by the parameter x, the size of the pulses Δv_n also depends on the orbit plane changes i_n $(n = 0,1,2)$.

From Fig. 2 it can be seen that

$$\frac{d\Delta v_n}{di_n} = v_{2n}\sin\alpha_{2n} = v_{2n+1}\sin\alpha_{2n+1}, \quad n = 0,1,2 \tag{3}$$

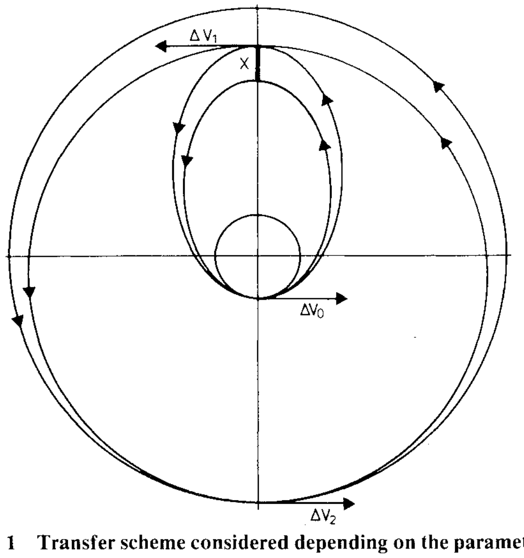

Fig. 1 Transfer scheme considered depending on the parameter x.

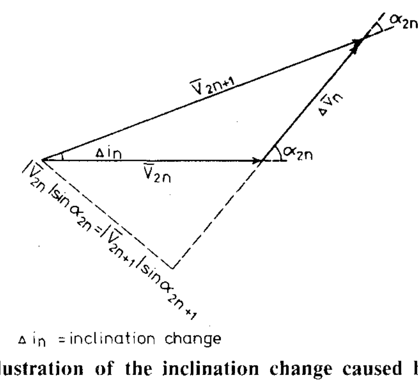

Fig. 2 Illustration of the inclination change caused by the pulses Δv_n, $n = 0,1,2$, orthogonal to radius vector.

where v_{2n} and v_{2n+1} are the velocities before and after pulse Δv_n.

A necessary condition for the minimum of $\Delta v_0(i_0) + \Delta v_1(i_1) + \Delta v_2(i_2)$ under the constraint $i_0 + i_1 + i_2 = i$, where i is the inclination between TO and SO, is therefore that

$$v_i \sin\alpha_i = \text{const}, \quad 0 \leq i \leq 5$$

where α_i are the angles indicated in Fig. 2.

From the fact that

$$v_3 \simeq v_4$$

it then follows that $\alpha_3 \simeq \alpha_4$. As the velocities $v_3 v_4$ are in opposite directions we conclude that while the first NSO pulse is made in the ABM firing direction with a bottom axial thruster, the second pulse can be made with a top axial thruster in the same attitude.

The size of the pulses Δv_0, Δv_1, and Δv_2 as functions of x are displayed in Fig. 3 under the assumptions that apogee of the initial TO is 3000 km below geostationary altitude, that the initial inclination between TO and SO is 28.8 deg (typical for launches from Eastern Test Range), and that for each x the partition of the inclination change $i_0 + i_1 + i_2 = 28.8$ deg is made to minimize

$$\Delta v_0(i_0) + \Delta v_1(i_1) + \Delta v_2(i_2)$$

It can be seen that Δv_0, Δv_1, and Δv_2 are almost linear functions of x except for the corner in the graph of Δv_2 for the value $x = 3000$ km where Δv_2 takes the value zero. $\Delta v_{10} = 1844$ m/s is the cost for the acquisition of SO for the nominal mission, so $\Delta v_0 + \Delta v_1 + \Delta v_2 - \Delta v_{10}$ represents the increase in necessary delta-v for the transfer into SO relative to the nominal mission. It can be seen that for $x = 0$ this delta-v is 142 m/s and that its minimal value of 52 m/s is taken for $x = 3000$ km corresponding to the optimal transfer discussed in the beginning of the paragraph.

The line $\Delta v_{\text{ABM}} - \Delta v_{10}$ intersects the line $\Delta v_1 - \Delta v_{10}$ for $x = 1600$ km. This corresponds to the case that the pulse from the ABM firing is equal to the pulse Δv_1, i.e., the ABM firing will lift perigee to geostationary altitude. This is clearly the largest x for which the transfer defined above can be implemented using the solid propellant ABM. It is seen from Fig. 3 that $\Delta v_0 + \Delta v_1 + \Delta v_2 - \Delta v_{10}$ takes the value 94 m/s to be compared to the value 142 m/s for the case of no apogee raising and the theoretical minimum of 52 m/s.

To the gain from the decrease in total delta-v should be added the increase in total propulsion capacity originating from the fact that hydrazine has been used before the ABM firing. This increase Δv_E is also plotted as a function of x. For

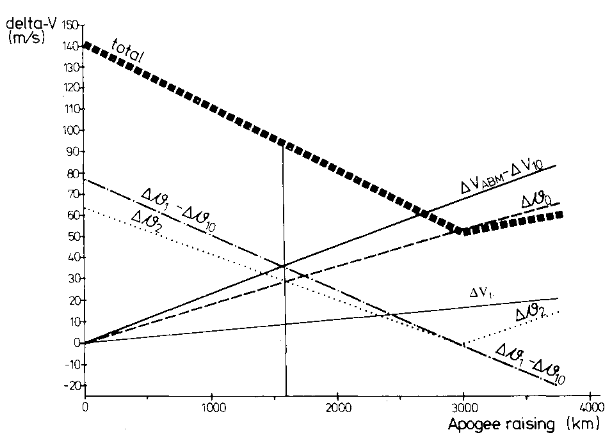

Fig. 3 Apogee raising with a perfect perigee impulse.

$x = 1600$ km the increase is 9 m/s, i.e., there will be $142 - 94 + 9$ m/s $= 57$ m/s more delta-v available in SO than if no apogee raising is undertaken.

It should finally be pointed out that as Δv_0, Δv_1, Δv_2 are essentially linear functions of x for apogee variations of the order of magnitude considered here, the results above generalize as follows:

1) The optimal amount of apogee raising is 1600/3000 = 53% of the initial apogee default.

2) The sizes of corresponding pulses Δv_0, Δv_1, Δv_2 and the total fuel saving are proportional to this default (the case of 3000-km apogee default being treated above).

Operational Implementation

When this transfer scheme is implemented operationally, constraints originating from the design of the RCS must also be taken into account. Typical constraints are:

1) no time tag facility, the apogee raising maneuver can only start when there is ground station coverage which is rarely available close to perigee; and

2) low thrust force, fairly long arcs of the orbit are needed for the execution of significant apogee raising maneuvers.

This means that in practice, the delta-v (i.e., the time integral of the acceleration caused by the thruster) that is needed for a certain apogee raising always will be larger than the Δv_0 plotted in Fig. 3 computed under the assumption of a perfect perigee impulse.

Considering a thrust direction aligned with the velocity vector at perigee Lagrange planetary equations for the apogee radius R_a, the perigee radius R_p and the rotation of the apsidal line w take the form

$$\dot{R}_a = a\frac{1+e}{1-e}\sqrt{\frac{p}{\mu}}\frac{(1+\cos\theta)^2}{1+e\cos\theta}f \tag{4}$$

$$\dot{R}_p = -a\frac{1-e}{1+e}\sqrt{\frac{p}{\mu}}\frac{(1-\cos\theta)^2}{1+e\cos\theta}f \tag{5}$$

$$\dot{w} = \frac{1}{e}\sqrt{\frac{p}{\mu}}\;\frac{\sin\theta\cos\theta}{1+e\cos\theta}f \tag{6}$$

where f is the acceleration caused by the thrust.

It is seen that with such a thrust direction R_a increases and R_p decreases for any true anomaly θ.

Assuming the maneuver to be executed over an arch symmetric around perigee, the time integrals of Eqs. (4-6) are

$$\Delta R_a = \frac{a^3\sqrt{1-e^2}}{\mu}\;(3E + 4\sin E + (1/2)\sin 2E)f \tag{7}$$

$$\Delta R_p = \frac{a^3\sqrt{1-e^2}}{\mu}\;(-3E + 4\sin E - (1/2)\sin 2E)f \tag{8}$$

$$\Delta w = 0 \tag{9}$$

where $-E$ and E are the eccentric anomalies for the start and the end of the maneuver. As the duration of the maneuver is

$$2\sqrt{a^3/\mu}(E - e\sin E) \tag{10}$$

one gets that

$$\frac{\Delta R_a}{\Delta v_0} = a\sqrt{\frac{p}{\mu}}\;\frac{(3/2)E + 2\sin E + (1/4)\sin 2E}{E - e\sin E} \tag{11}$$

$$\frac{\Delta R_p}{\Delta v_0} = a\sqrt{\frac{p}{\mu}}\;\frac{-(3/2)E + 2\sin E - (1/4)\sin 2E}{E - e\sin E} \tag{12}$$

Equations (11) and (12) are plotted in Figs. 4 and 5 as functions of true anomaly.

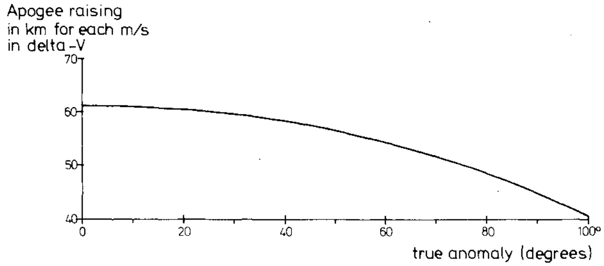

Fig. 4 Efficiency of an apogee raising maneuver symmetric about perigee as a function of true anomaly of maneuver end.

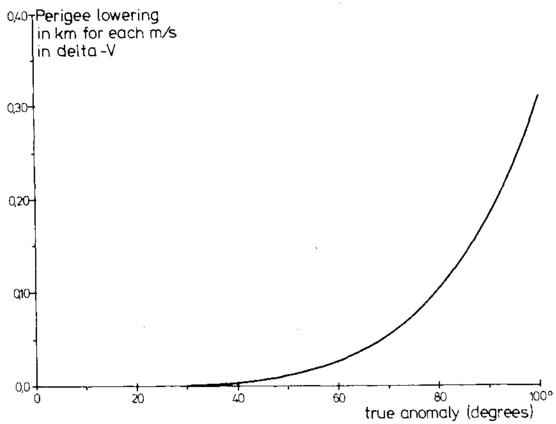

Fig. 5 Perigee lowering caused by an apogee raising maneuver symmetric about perigee as a function of true anomaly for maneuver end.

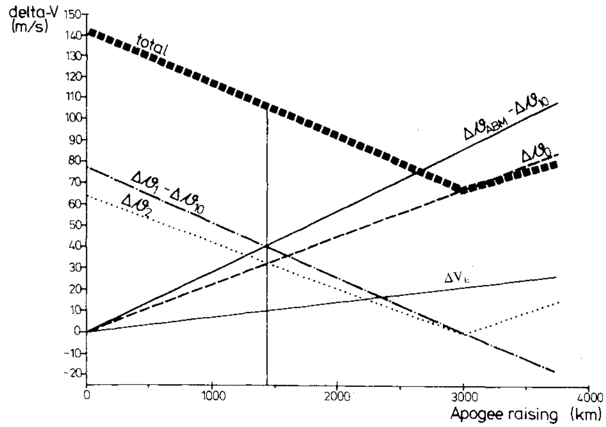

Fig. 6 Apogee raising with maneuvers extending from −90 to 90 deg in true anomaly.

Available analyses of the coverage situation with the Space Flight Tracking and Data Network (STDN) ground station network of NASA give the result that there will always be enough orbits with ground station coverage somewhere between perigee and true anomaly −90 deg. Apogee raising maneuvers can then be initiated from such a ground station and go on to the other side of perigee. By making one or several such maneuvers it will be possible to make Δv_I and Δv_{ABM} coincide, the efficiency of the apogee raising being as displayed in Fig. 4.

The resulting perigee lowering can be read from Fig. 5. It is seen that the perigee lowering caused by a maneuver not extending beyond ±90 deg true anomaly is negligible from all points of view except possibly in some extreme cases for airdrag. The modifications of Fig. 3 assuming the apogee raising maneuver to be executed between −90 and 90 deg true anomaly (worst case) are displayed in Fig. 6. Δv_0 and Δv_{ABM} now increases more rapidly as functions of the apogee raising x. Δv_{ABM} is equal to Δv_I for $x = 1410$ km and it is seen that the total delta-v for this transfer is 106 m/s larger than Δv_{I0}. The increase in propulsion capacity is 10 m/s so that the remaining propulsion capacity in SO is 11 m/s smaller than for the case of Fig. 3 with an impulsive maneuver at perigee but still 46 m/s larger than if no apogee raising is undertaken.

Conclusions

In case a launcher anomaly causes a geostationary spacecraft to be injected into a transfer orbit with apogee significantly below nominal, an apogee raising as outlined in this paper will result in a saving in hydrazine corresponding to about 15-19 m/s for each 1000 km the apogee is below nominal.

To achieve this, the apogee should be raised so much that the apogee boost motor firing will lift perigee to geostationary altitude because of the combined effect of the higher apogee *and* the lower mass.

References

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[2] Gobetz, F.W. and Doll, J.R., "A Survey of Impulsive Trajectories," *AIAA Journal*, Vol. 7, May 1969, pp. 801-834.